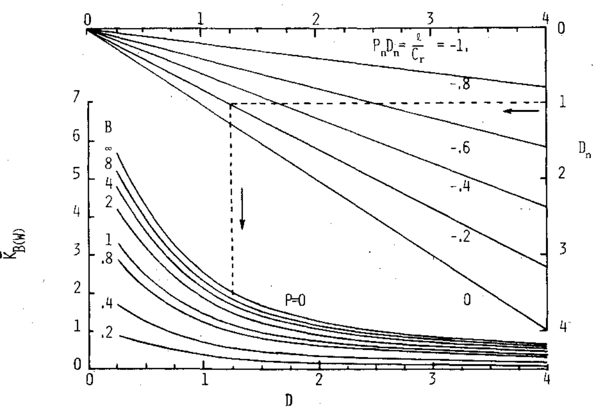

Fig. 2 Design chart to determine $\bar{K}_{B(W)}$ for negative afterbody from $\bar{K}_{B(W)}$ for no afterbody.

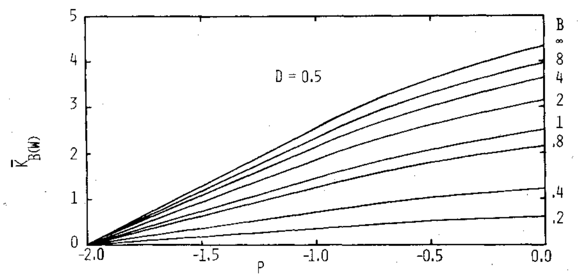

Fig. 3 $\bar{K}_{B(W)}$ for negative afterbody as a function of afterbody length. (If you relabel the abscissa as $R=P+2$, this figure is applicable for all P and D with $0 \leq R \leq 2$.)

upper-half straight line from specified values of D_n and ℓ/C_r, $\bar{K}_{B(W)}$ then can be looked up from a lower-half curve for a given B. For example, Fig. 2 yields $\bar{K}_{B(W)} \approx 2$ and $D \approx 1.25$ for $D_n = 1$, $\ell/C_r = -0.2$, and $B \rightarrow \infty$.

Variation of $\bar{K}_{B(W)}$ with respect to afterbody length is illustrated in Fig. 3 for the full range of B values. Note that for a fixed B the dependence of $\bar{K}_{B(W)}$ on R is linear if R is less than or equal to unity.

Conclusion

Closed-form formulas have been presented for the interference factors $\bar{K}_{B(W)}$ for a body in the presence of a wing at supersonic speeds when a finite afterbody is introduced. In addition, formulas for $\bar{K}_{B(W)}$'s have been obtained for situations when the base of the body is forward of the trailing edge of the exposed root chord (negative afterbody). These formulas are valid subject to the restrictions inherent in the formulation of Ref. 2.

Remarks

Similar to $\bar{K}_{B(W)}$, the body longitudinal center-of-pressure location can be determined in closed form. This work is currently in progress and will be reported later.

Acknowledgments

Support of this work by NASA Contract NAS5-24242 to Howard University is hereby gratefully acknowledged. The authors wish to express their thanks to the technical officer of the contract, Mr. James S. Barrowman, Code 742, at the NASA Goddard Space Flight Center in Greenbelt, Md.

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AIAA 82-4147

Calculation of Unsteady Transonic Flows with Shocks by Field Panel Methods

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Introduction

To date the various successful finite difference methods (FDM)[1,2] to investigate unsteady inviscid transonic flow have almost overshadowed other methods. Although the computation times of the versions for two-dimensional flow are reasonable, the prospects for three-dimensional flow are unattractive. At NLR, therefore, activities were begun a few years ago to develop competitive integral equation methods (IEM). For steady flow this work has resulted in the NLR field panel method (FPM),[3] and improvements are being attempted.[4] The purpose of this Note is to indicate the status of developments for unsteady flow.

IEM Formulation

First the basic equations for an unsteady IEM are briefly described. The boundary-value problem that governs the unsteady "time-linearized" inviscid transonic potential flow past a thin two-dimensional wing at a small angle of attack is shown in Fig. 1. There, M is the freestream Mach number; k the reduced frequency, $k=\omega c/2U$, where c denotes the chord; and U the undisturbed flow speed. $\Phi^{(0)}$ and $\Phi^{(1)}$ denote the mean steady and the first harmonic component of the perturbation velocity potential, and H the amplitude of oscillation of the mean wing surface. γ^* is defined[1] by: $\gamma^* = 2-(2-\gamma)M^2$. The variables are all made dimensionless with c and U. The pressure coefficient $C_p^{(1)}$ is made dimensionless with the dynamic pressure. An exact solution of this boundary-value problem will show a jump in $\Phi^{(1)}$ across the steady-shock location which is proportional to the integrated pressure coefficient inside the shock trajectory.[5] A solution of the boundary-value problem can be expressed by:

$$\Phi^{(1)}(x,y) = \int_0^\infty \Delta\Phi^{(1)}(u)\frac{\partial}{\partial y}E(x-u,y)\,\mathrm{d}u + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m(u,v)E(x-u,y-v)\,\mathrm{d}u\mathrm{d}v \quad (1)$$

where $E(x,y)$ satisfies the radiation condition and

$$(1-M^2)E_{xx} + E_{yy} - 2ikM^2E_x + k^2M^2E = \delta(x)\delta(y) \quad (2)$$

Received July 6, 1981; revision received Nov. 2, 1981.

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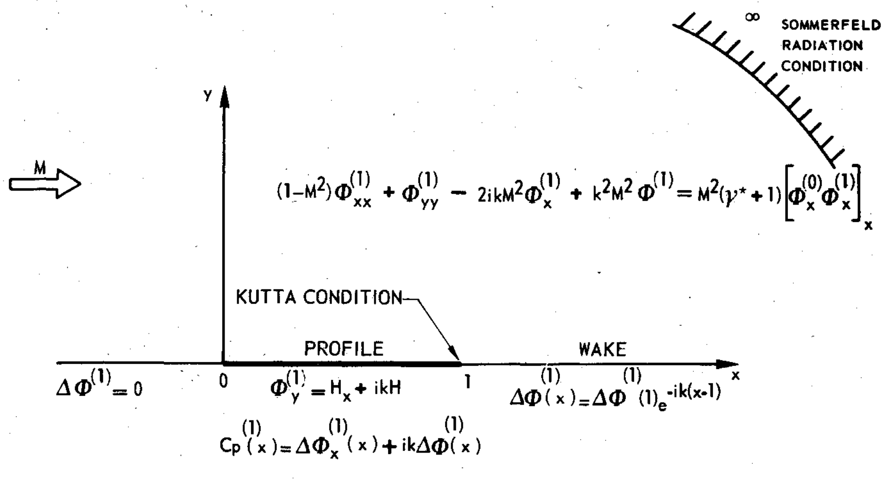

Fig. 1 The boundary-value problem.

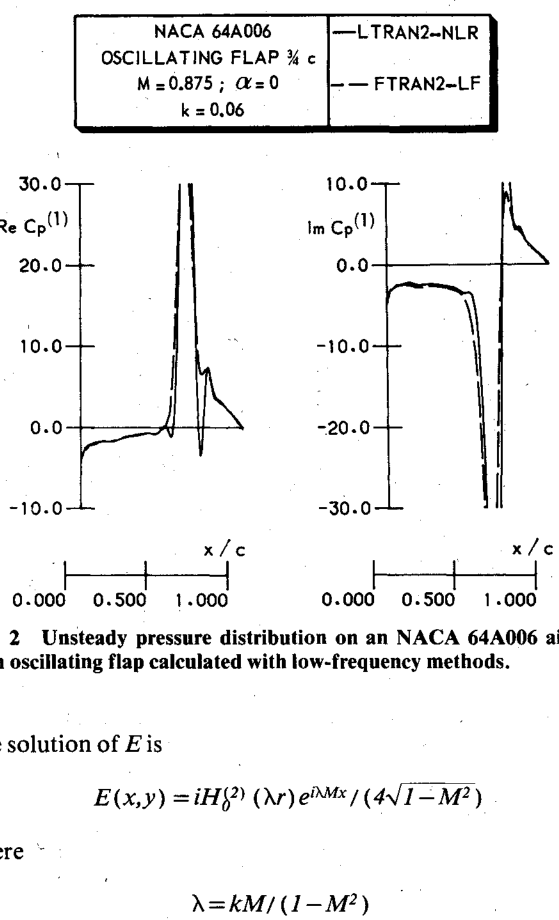

Fig. 2 Unsteady pressure distribution on an NACA 64A006 airfoil with oscillating flap calculated with low-frequency methods.

The solution of E is

$$E(x,y) = iH_0^{(2)}(\lambda r)e^{i\lambda Mx}/(4\sqrt{1-M^2})$$

where

$$\lambda = kM/(1-M^2)$$

and

$$r = \sqrt{x^2 + (1-M^2)y^2}$$

$m(x,y)$ is a source strength distribution and $\Delta\Phi^{(1)}$ is a dipole strength distribution. After substitution of Eq. (1) into the TSP equation given in Fig. 1, it follows that

$$m(x,y) = (\gamma^* + 1)M^2[\Phi_x^{(0)}\Phi_x^{(1)}]_x \tag{3}$$

In this way the IEM formulation is given by Eqs. (1-3) and by the boundary conditions at the airfoil and wake. $\Phi_x^{(0)}$ should be known from a previous steady calculation. It is important to realize that the radiation condition is implicitly satisfied and that the computational domain can be restricted to the part of the flow where nonuniform compressibility effects are important.

Description of FTRAN2

Recently a computer code, FTRAN2, has been developed at NLR which solves the above boundary-value problem. It is a field panel method (FPM) that approximates the integrals in Eq. (1) at selected points in the field (field points) and at the chord (chord points) by a summation of weight factors (influence coefficients) times the potential jump at the chord points and the source strength at the field points. In the present analysis the field points are chosen in the midpoints of the field panels obtained by choosing a rectangular, equidistant grid in the x-y plane ($\Delta x = h$, $\Delta y = h/\sqrt{1-M^2}$). The chord points are chosen in the midpoints of the chord panels obtained by subdividing the chord into equidistant elements. The set of chord points is completed with the point at the trailing edge (wake point) which determines $\Delta\Phi^{(1)}$ in the wake. The weight factors are determined in such a way that the approximations have an accuracy of $\mathcal{O}(h^2)$.

Equation (3) is discretized in the following way:

$$m_{ij} = D_x[(\gamma^* + 1)M^2\Phi_{x_{ij}}^{(0)}\Phi_{x_{ij}}^{(1)} - (1-M^2)\Phi_{x_{ij}}^{(1)}] + (1-M^2)\delta_x^2\Phi_{ij}^{(1)} \tag{4}$$

where D_x encompasses the well-known Murman's[6] conservative mixed-difference operator. δ_x is the central difference operator. With the introduction of D_x the discontinuities at the steady-shock location are captured as $\mathcal{O}(h)$ narrow regions with continuous but high velocities and pressures. The source and dipole distributions are found by specifying the tangential flow condition compatible with the wing motion at the chord points, the Kutta condition at the wake point, and Eq. (4) at the field points. The resulting system of linear algebraic equations is solved by a block-iterative method. The main difference between the present formulation and similar approach of Nixon[7] is that the latter is more complicated and includes boundary conditions to be applied at the shock location.

Calculations with FTRAN2

Calculations of unsteady pressure distributions have been performed with the FDM code LTRAN2-NLR[1] and the FTRAN2-LF code for the NACA 64A006 airfoil for flap rotation about the flap leading edge at the three-quarter chord in a flow with $M = 0.875$ and $\alpha = 0$ deg ($k = 0.06$). FTRAN2-LF is a low-frequency version of FTRAN2 in which the term $k^2M^2\Phi^{(1)}$ in the TSP equation has been omitted in order to be fully compatible with the LTRAN2-NLR method. The steady-flowfield data have been provided by the LTRAN2-NLR code. Figure 2 shows good agreement between the codes. The differences that still exist at the pressure peaks have no quantitative significance to the lift and moment coefficients.

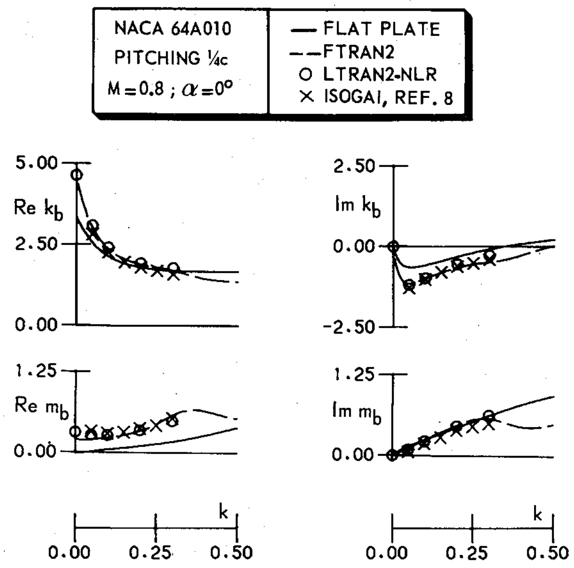

Fig. 3 Unsteady airloads on a pitching NACA 64A010 airfoil.

Further, unsteady-load calculations have been performed with FTRAN2 and LTRAN2-NLR for the NACA 64A010 airfoil pitching about the quarter chord in a flow with $M=0.8$ and $\alpha=0$ deg. The steady-flowfield data have been obtained again with LTRAN2-NLR. In Fig. 3 unsteady lift and moment are shown which have been put into standard AGARD notation ($k_b=C_\ell^{(1)}/\pi H_x$, $m_b=2C_m^{(1)}/\pi H_x$). Also shown are results of the FDM presented in Ref. 8 and flat plate results. The transonic codes agree very well. The FTRAN2 results have been obtained within 25% of the CPU time needed for the LTRAN2-NLR calculations.

Modified FTRAN2 Formulation

The computation time of FTRAN2, which is proportional to N^2 with N being the number of unknowns, should be reduced further to enable an extension of FTRAN2 to three-dimensional applications. To that purpose a modification of FTRAN2 is proposed in this section.

Consider the following approximation of the elementary solution of Eq. (2):

$$E(x,y) = \sum_{m=1}^{M} E_m(x,y) \tag{5}$$

Then $\Phi^{(1)}$ can be expressed accordingly as

$$\Phi^{(1)} = \sum_{m=1}^{M} \Phi_m^{(1)} \tag{6}$$

where $\Phi_m^{(1)}$ is defined by Eq. (1) when E is replaced by E_m.

The essence of introducing Eq. (5) is that the set E_m is chosen in such a way that each $\Phi_m^{(1)}(x,y)$ should be a smoother function than $\Phi_{m-1}^{(1)}(x,y)$. The property "smooth" should be explained here as follows: If each $\Phi_m^{(1)}$ is approximated by a Fourier series, then the small wavelength contributions should decrease with increasing m. The success of the proposed modification lies in a computational effort for each $\Phi_m^{(1)}$ which decreases with increasing m.

At present the following set E_m is evaluated at NLR:

$$E_m(r,\varphi) \equiv E(r,\varphi), \quad R_{m-1} \le r \le R_m$$

$$E_m(r,\varphi) \equiv 0, \qquad r>R_m \text{ and } 0<r<R_{m-1} \tag{7}$$

where $R_0=0$, $R_M=\infty$, r and φ being polar coordinates. The modified procedure is illustrated here for the $M=2$ case. $\Phi_1^{(1)}$ is determined as in FTRAN2 in all points, which leads approximately to $\mathcal{O}(NR_1^2/h^2)$ operations, in which $R_1^2/h^2 \ll N$. For determining $\Phi_2^{(1)}$ the following accelerated procedure is suggested:

1) Determine $\Phi_2^{(1)}$ as in FTRAN2 in a number N_c of specially selected points (collocation points), with $N_c \ll N$, which leads approximately to $\mathcal{O}[NN_c]$ operations.

2) Determine $\Phi_2^{(1)}$ in the other points by series, expansions, polynomials, or interpolation. The N_s coefficients of these series, expansions, etc., with $N_s \ll N$ are easily determined by collocation in the collocation points. Supposing $N_c=N_s$, this part requires approximately $\mathcal{O}[NN_c+N_c^3]$ operations.

The accelerated procedure has been applied already to the oscillating flat plate with and without control surface (no source distribution) using either Fourier series or Lagrange interpolation functions. The main objective was to find a suitable minimum value of R_1, N_c, and N_s. A factor $\sqrt{x}$ has been applied to the approximating series and the interpolation functions to account for the leading-edge behavior. In other trial calculations also, a small leading-edge region has been separated. It has turned out as sufficient to choose $R_1/h = \frac{1}{2}\sqrt{N}$, $N_c=N_s=\sqrt{N}$ (N here equals the number of chord points). The computational effort was proportional to $\mathcal{O}(N^{1.5})$, which is significantly less than $\mathcal{O}(N^2)$ of the FTRAN2 method.

Conclusions

The status of investigations of field panel methods for the "time-linearized" unsteady transonic flow problem at NLR has been described. Calculations with the computer code FTRAN2 for a NACA 64A006 and a NACA 64A010 airfoil yielded the following experience:

1) Calculated results compare well with those of well-established finite difference methods, e.g., LTRAN2-NLR.

2) The CPU time necessary in routine calculations is about 25% of the time needed by LTRAN2-NLR.

Finally, a modification of the field panel method formulation has been presented which implies an accelerated procedure in order to further reduce computational effort and make the method attractive to three-dimensional flow. Application to the flat plate problem led to the significant acceleration of $N^{1/2}$ where N is the number of unknowns.

Acknowledgment

This investigation was carried out under contract for the Scientific Research Division of the Directorate of Materiel, Royal Netherlands Air Force (RNLAF).

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